Angular Momentum Loss by Gravitational Radiation in Relativistic Binary Stars

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1.Introduction

Relativistic binary systems are key binaries to understand the end of the stellar evolution and also give us the opportunity to test the theory of general relativity. We have collected the physical parameters of some X-ray binary stars, binary systems with a black hole (BH)/neutron star (NS) / white dwarf (WD) companion and binaries with two degenerate companions (NS+NS, NS+WD, and WD+WD). We discuss the angular momentum loss mechanisms using this data set. We have calculated the angular momentum loss rates and the angular momentum loss time scales of the selected relativistic binaries, as well as very short period massive binary stars. We describe these as,

HMXRB-BH: High-Mass X-Ray Binary with a BH. A BH primary and a massive secondary component.
HMXRB-NS: High-Mass X-Ray Binary with a NS. A NS primary and a massive secondary component.
LMXRB-BH: Low-Mass X-Ray Binary with a BH. A BH primary and a low mass secondary component.
LMXRB-NS: Low-Mass X-Ray Binary with a BH. A NS primary and a low mass secondary component.
DDNS: Double degenerate NS binary system.
DDNSWD: Double degenerate NS with a WD secondary.
DDWD: Double degenerate WD binary system.
AMCVn : Double degenerate white dwarfs where a white dwarf primary accretes matter from a helium rich secondary.
NLCV: A white dwarf primary and a late type secondary component.
mCV: A *magnetic* white dwarf primary and a late type secondary component.
PreCV: Pre-cataclysmic variable systems(V471 Tau type). A WD and a RD.
LTCBs: Low temperature *contact* binary systems. Two late type stars.
HTCBs: High temperature *contact* binary systems. Two early type stars.

2. Data selection

In the last decade, numerous papers have been published on the determination of orbital and physical parameters of binaries and their degenerate and non-degenerate components. Observations done with new instruments attached to the big telescopes and the use of modern data reduction and modelling techniques lead to more accurate parameters. Thus we have selected relativistic and non-relativistic binaries whose components have reliable mass values. We summarize the mass range for different type of systems in Table1. The mass range of isolated black holes, neutron stars and white dwarfs are also given for the sake of comparison.

3. Angular Momentum

In close binaries, angular momentum can be studied in the form of orbital and spin. The spin angular momentum for a single star is given in the form,

$$H_i^s = I\omega = k^2 R_i^2 M_i \omega_i, \qquad (1)$$

where I is the moment of inertia and \$\omega\$ is the spin angular velocity, k^2 is the gyroton radius, R is the radius of the star and M is the mass of the star. Similarly the total orbital angular momentum is,

$$H_t^o = M_1 M_2 \sqrt{\frac{Ga(1-e^2)}{M}} = 111q(1+q)^{-2} M_t^{-5/3} P^{1/3} \sqrt{1-e^2}\ (M_\odot R_\odot^2/day) \qquad (2)$$

where, G is gravitational constant, a is the separation of two stars and q is the mass ratio of the components. The angular momentum is extracted from the binary orbit by (i) magnetic braking via a magnetized stellar wind (MSW), (ii) gravitational radiation (GR), (iii) non-conservative mass transfer and (iv) a third (or multi) body effect. The system's angular momentum loss is the sum of all these effects.

Here we assume that the relativistic binary evolution is driven by GR and MSW.

MSW

The gas located at the equator of the Sun is known to rotate more rapidly than the gas at the poles. This is known as differential rotation. Differential rotational of a star is crucial in generating magnetic field in the convective zone. Many close binaries show an asymmetry in their light curves where one maximum is higher than the other (the O'Connell effect). These asymmetries are usually attributed to magnetic activity. Magnetic activity effects have been observed many times in late type stars. This mechanism may also play an important role in relativistic binaries whose components are late type stars. Relativistic binaries with late type (M < 1.5 M_{sun}) components can carry angular momentum from the system in a magnetized stellar wind.
According to Kraft, Skumanich and Smith, solar-type main-sequence stars seem to exhibit a strong correlation between equatorial rotation velocity and age. This correlation can be expressed in the form, $v_e = 10^{14}$x f x $t_o^{-0.5}$ cm s^{-1}, where t_o is the age of the star and f is a constant range between 0.75 - 1.75. Starting with the Skumanich law and using Eq.(1) we can deduce the angular momentum loss rate.

$$\left(\frac{dH}{dt}\right)_{\rm MSW} = -1.6 \times 10^{-30} M_2 R_2^4 \omega^3. \qquad (3)$$

The angular momentum time scale in Gyr is

$$\tau_{\rm MSW} = -14 M^{2/3} R_2^{-4} P^{10/3} (1+q)(1-e^2)^{1/2}. \qquad (4)$$

To calculate the radii of secondary stars we assume R_2=R_L (Eggleton 1983), if they filled their Roche lobes, otherwise we used the mass-radius relations given by Yakut (2008b).

GR

The shape of the binary orbit and the energy of the system is given by

$$r = \frac{a(e^2-1)}{1+ecos\nu} \quad \text{and} \quad E = \frac{-GM_1M_2}{2a}. \qquad (5)$$

Using these equations and the semi-major axis change rates (Landau & Lifshitz) we can derive the angular momentum loss by gravitational radiation and its time scale in Gyr, after some straightforward steps as,

$$\frac{da}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\frac{M_1M_2}{a^3}(M_1+M_2), \qquad (6)$$

$$\left(\frac{dH}{dt}\right)_{GR} = -3.44 \times 10^{44} (M_1 M_2 M^{-1/3})^2 P^{-7/3} f(e) \qquad (7)$$

$$\tau_{\rm GR} = 376.4 q^{-1}(1+q)^2 M^{-5/3} P^{8/3} (1-e^2)^{3/2} (1+\frac{7}{4}e^2)^{-1} {\rm Gyr} \qquad (8)$$

We show the angular momentum loss time scales for relativistic and non-relativistic binaries in Figures. The details will be discussed in Yakut et al. (2008a).

Table 1. Observational mass range of relativistic binaries and isolated compact stars

Type	pri+sec	component n	M_1 max.	min.	M_2 max.	min.
HMXRB	BH+S	6	23.1	6.0	70	6.5
HMXRB	NS+S	11	2.4	1.0	58	8.8
LMXRB	BH+S	13	14	4.0	2.7	0.37
LMXRB	NS+S	6	1.8	1.4	2.3	0.4
NLCV	WD+S/B	39	1.4	0.3	1.1	0.05
mCV	WD+S	10	0.8	0.4	0.5	0.1
PreCV	WD+S	21	0.84	0.39	0.93	0.1
DDNSNS	NS+NS	8	1.6	1.14	1.4	1.05
DDNSWD	NS+WD	9	2.1	1.27	1.3	0.16
DDWDWD	WD+WD	10	0.7	0.32	0.7	0.29
AMCVn	WD+WD	5	0.98	0.59	0.13	0.011
Black Hole		19	23.1	4.0		
Neutron star		42	2.4	1.0		
White Dwarf		90	1.4	0.01		

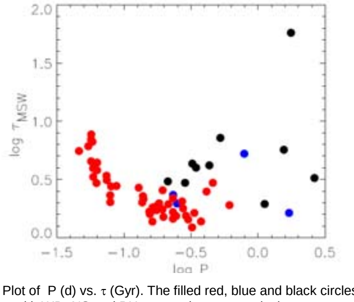

Fig 1. Plot of P (d) vs. τ (Gyr). The filled red, blue and black circles show binaries with WD, NS and BH companions respectively.

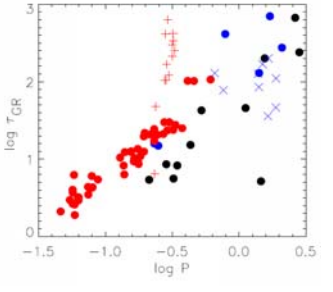

Fig 2. Plot of P (d) vs. τ_{GR} (Gyr). The filled red, blue and black circles show binaries with WD, NS, and BH companions and + and x show the LTCB and HTCB systems, respectively.

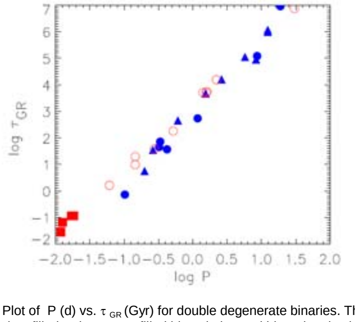

Fig 3. Plot of P (d) vs. τ_{GR} (Gyr) for double degenerate binaries. The open red circles, filled red squares, filled blue circles and blue triangles indicate WD+WD, WD+WD (AM CVn), NS+NS and NS+WD binaries, respectively.